

# Design Problem Set 1

## *Steady State and Dynamic Analysis of a Process Control System*

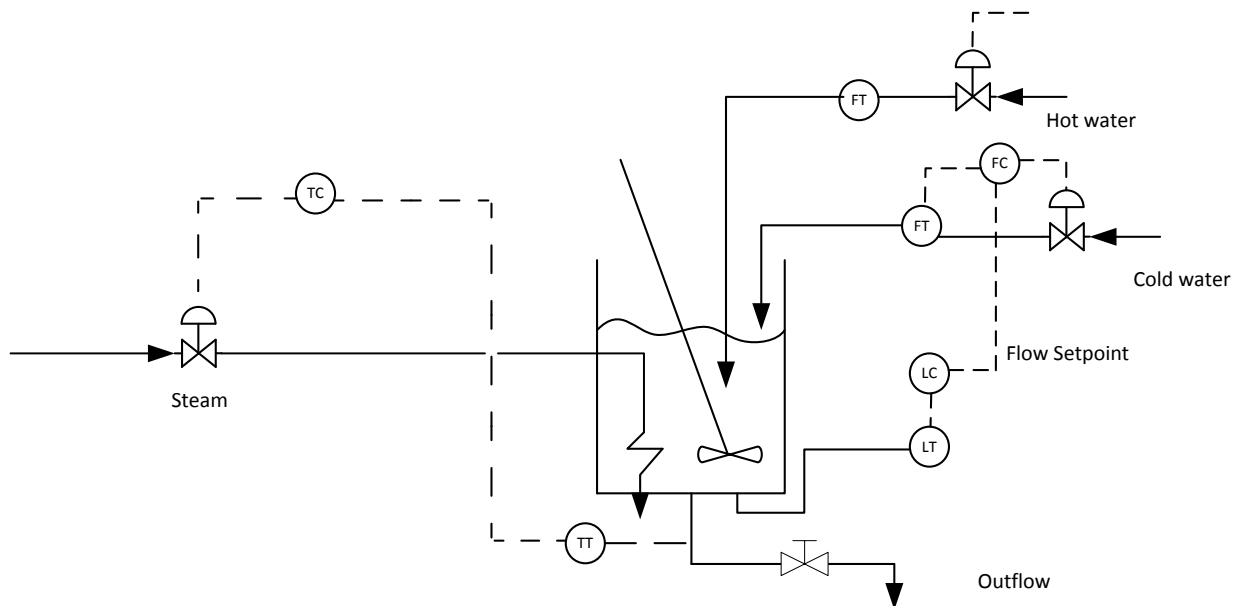
### Objectives

The objective of this problem set is to set up a linear input-output model of a process, implement a P-only controller structure, and examine the steady state and transient performance in reference tracking, disturbance rejection, and variation of operating point.

Answers are to be recorded directly in the spaces provided. Please attach additional pages, if required.

### Instructions

Below is a P&ID<sup>1</sup> description of a steam heated stirred tank process.



<sup>1</sup> Please refer to the Appendix for Piping and Instrumentation Diagram (P&ID) symbols.

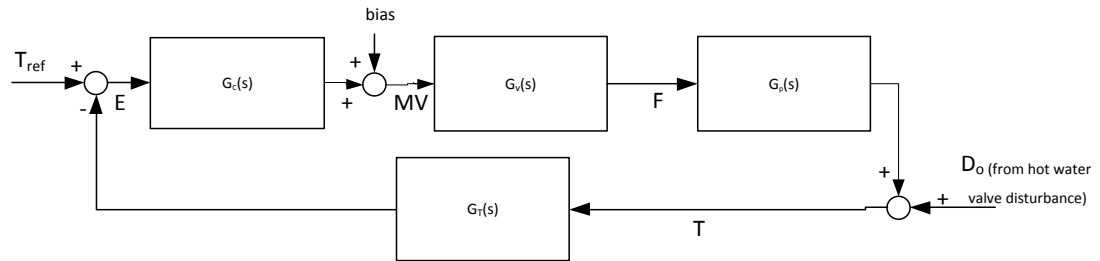
1. There are three control valves shown, each pneumatically controlled via a current to pressure transducer. Briefly describe what each control valve does. I.e., what is the physical signal that it is being managed by the valve.

Control Valve	What does it do?
1.	
2.	
3.	

2. What control loops are present and briefly describe the control objective

Control Loops Present	Control Objective
1.	
2.	
3.	

3. Consider the temperature control loop, shown in block diagram form below:



The hot water valve is closed and the automatic level controller is maintaining a constant water level at approximately 3 metres in a tank that is 6 metres in height. Suppose that a nominal operating point of 55 deg. C. is chosen. To achieve this, the control valve has been initially adjusted in manual mode so that it is 25% open. The controller produces a 4-20 mA output. The steam valve is presumed to exhibit a linear behaviour, and can deliver a maximum of 30 kg/min. steam flow. The temperature transmitter is modelled as unity gain. The tank temperature changes by 2 deg. C. in response to a 1 kg/min change in steam flow.

Determine the gains of the valve ( $K_V$ ) and process ( $K_P$ ), making sure to indicate the engineering units used. Parameterize all 7 signals shown in the block diagram with their steady state values expressed in engineering units (i.e., not percent of scale). Show your work below:

**Note:** All questions following will be with respect to this steady state operating point.

4. Our plan is to determine if a linear controller is an appropriate choice for temperature control. To do this, we'll characterize the controller's ability to provide

- Good tracking of the reference temperature (as defined next in 5.)
- Rejection of potential hot water disturbances
- Robustness to variation of operating point

The first step is to work out the transfer functions, with respect to the steady state operating point defined in 3., from  $T_{ref}(s)$  to  $T(s)$  and from  $D_o(s)$  to  $T(s)$ . Use either block diagram reduction or signal flow graph theory to do this. Leave the transfer functions representations as  $G_C(s)$ ,  $G_V(s)$  and  $G_P(s)$  (i.e., don't attempt to parameterize them just yet). Attach all calculations separately to this assignment. Show the final transfer functions below:

5. With the information gathered so far, determine the **minimum** proportional gain  $K_C$  to be used to ensure that the maximum steady state error to a 10 deg. step change in  $T_{ref}$  is at most  $0.X$  deg. 'X' is the last digit of your student number (unless that digit = 0, then 'X=1'). For example, if your student number ends in 7, then  $X=7$ , and you are looking for the minimum proportional gain that will ensure that  $T$  differs from  $T_{ref}$  by at most 0.7 degrees. Show your work in the space below.

6. The reference temperature is returned back to 55 deg. C., and steady state is resumed. Given all the information gathered so far, with proportional controller as designed in step 5., what is the **maximum** hot water temperature disturbance that you can handle to keep within  $0.X$  deg. of the reference temperature. Show your work in the space below, and don't forget the units.

7. Suppose that the valve has a first order transfer function with time constant of 4 seconds, the transmitter has a first order filter with time constant of 0.2 seconds, and the tank is modeled at the operating point as a first order with time constant of 40 seconds. The controller is proportional-only, with gain as computed in 5. Use this information along with the additional gains computed in 3. to parameterize each of the 4 transfer functions for the blocks shown in the block diagram. Show your work in the space below.

8. If the level controller were to experience a set point change, the gain of the process,  $K_P$ , is expected to change as well. Compute the steady state sensitivity of the closed loop transfer function to variation in process gain, i.e.,  $S_{K_P}^{G_{cl}}$ , which is the ratio of percent change in closed loop transfer function to percent change in process gain. Use the proportional controller gain that you selected in 5. in completing the calculation Show your work in the space below:

9. Given the sensitivity calculation in 8., and supposing that each 10 cm increase in tank level decreases the process gain  $K_P$  by 5%, what is the maximum level change in cm. that can be tolerated to ensure that the steady state change (complex frequency  $s \approx 0$ ) in the closed loop transfer function gain is kept to 2%? Show your work in the space below:

10. With the proportional gain as designed in 5., and transfer functions as specified in 7., come up with a reduced (2<sup>nd</sup>) order model that will adequately represent the closed loop dynamics. Show your work in the space below:



11. What is the maximum value, in deg. C., based on your reduced order model, you will expect to see in response to a 10 deg. C. step change in reference temperature? Show your work in the space below:

12. What is the 0-100% rise time and 2% settling time, in seconds, based on your reduced order model, will you expect to see in response to a 10 deg. C step change in reference temperature? Show your work in the space below:

13. Use the MATLAB function `stepval.m` (available on Share Out) to plot the response to a 10 deg. C step change in reference temperature for your 3<sup>rd</sup> order model, record how much overshoot, in deg. C. 0-100% rise time in seconds, and 2% settling time in seconds, and compare the difference in these results with **your calculations from 11. and 12.** which were based on a 2<sup>nd</sup> order model of the closed loop system dynamics. Include the plot of the 3<sup>rd</sup> order system response to the 10 deg C. step away from reference temperature here:

Record your results below:

	%OS	$T_{rise,0-100\%}$ (sec)	$T_{s,2\%}$ (sec)	% change in Overshoot  2nd – 3rd  / 3rd *100%	%change in $T_{rise,0-100\%}$  2nd – 3rd  / 3rd *100%	%change in $T_{s,2\%}$  2nd – 3rd  / 3rd *100%
3 <sup>rd</sup> order model						
2 <sup>nd</sup> order approximate model						

14. This process is expected to operate at a reference temperature of  $55 \pm 5$  deg. C., be subjected to variations of  $\pm 0.25$  metres away from reference level of 3 metres, and experience hot water disturbances of up to 5 deg. C. Reflecting on the plan outlined in 4., and given the offset error specification in 5., is your linear proportional-only controller a reasonable choice for this application? Justify your opinion below by performing a 'worst case' analysis of the overall steady state and transient performance.

## Appendix

Normalized 2<sup>nd</sup> order linear transfer function:  $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

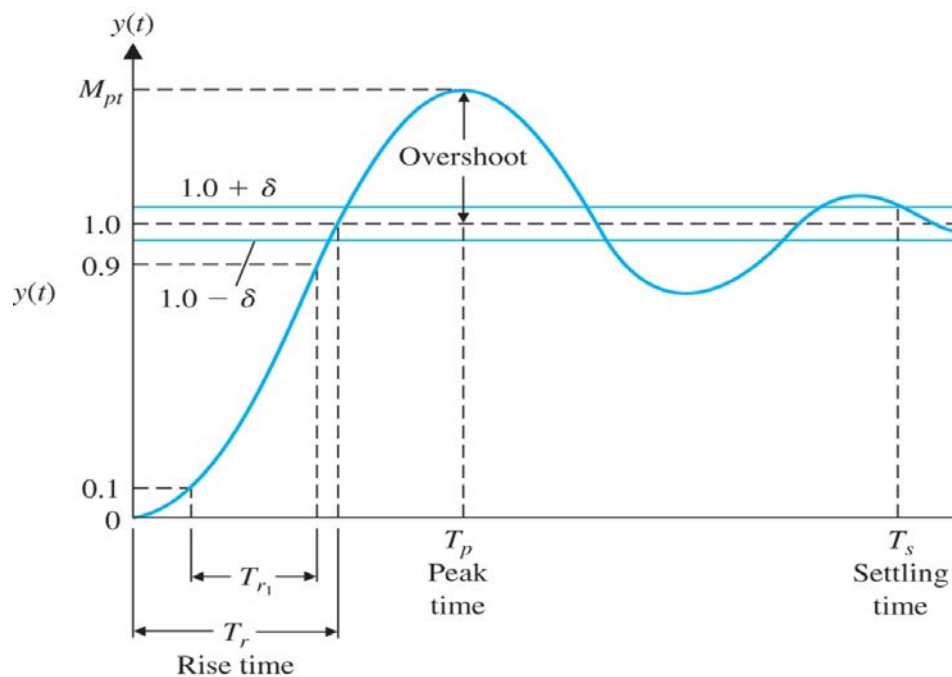
$$\%OS = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_{pt} = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$T_{s,2\%} = \frac{4}{\zeta\omega_n}$$

$$T_{r1} \approx \frac{2.16\zeta + 0.60}{\omega_n}; 0.3 < \zeta < 0.8$$



Normalized second order linear system unit step response

### Piping and Instrumentation Symbols

