Assignment #4

1) An image has defined as $f(x,y) = \sin(2x+3y)$, so the Fourier transform of this image is $F(u,v)$

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin(2x+3y)\,e^{-j2\pi(ux+vy)}\,dx\,dy =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2j}[\,e^{j(2x+3y)} - e^{-j(2x+3y)}]\,e^{-j2\pi(ux+vy)}\,dx\,dy = \frac{1}{2j}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j(2x+3y)}\,e^{-j2\pi(ux+vy)}\,dx\,dy -$$

$$\frac{1}{2j}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-j(2x+3y)}\,e^{-j2\pi(ux+vy)}\,dx\,dy = \frac{-j}{2}[\delta(u+\frac{2}{2\pi}, v+\frac{3}{2\pi}) - \delta(u-\frac{2}{2\pi}, v-\frac{3}{2\pi})]$$

2) If an image f(x,y) is padded with zeros as g(x,y), the difference appear in F(u,v) and G(u,v) are

1- The average of g(x,y) is smaller than the average of f(x,y). since F(0,0) and g(0,0) are showing the average value of f(x,y) and g(x,y) expectedly, so G(0,0)< F(0,0)
2- Padding with zeros will introduce discontinuities at the boarders and strong horizontal and vertical edges. This produces stronger spectrum along horizontal and vertical axes of the spectrum

3) The filter transfer function H(u,v) for a filter that averages the four immediate neighbors of a point (x,y) , but does not include f(x,y) can be found as

$$g(x,y) = \frac{1}{4}[f(x,y+1) + f(x+1,y) + f(x-1,y) + f(x,y-1)]$$

$$G(u,v) = \frac{1}{4}[F(u,v)e^{\frac{j2\pi v}{N}} + F(u,v)e^{\frac{j2\pi u}{M}} + F(u,v)e^{-\frac{j2\pi u}{M}} + F(u,v)e^{\frac{-j2\pi v}{N}}]$$

$$G(u,v) = \frac{F(u,v)}{4}[e^{\frac{j2\pi v}{N}} + e^{\frac{j2\pi u}{M}} + e^{-\frac{j2\pi u}{M}} + e^{\frac{-j2\pi v}{N}}]$$

$$G(u,v) = F(u,v).H(u,v)$$

$$H(u,v) = \frac{1}{4}[e^{\frac{j2\pi v}{N}} + e^{\frac{j2\pi u}{M}} + e^{-\frac{j2\pi u}{M}} + e^{\frac{-j2\pi v}{N}}] = \frac{1}{2}[\cos(2\pi\frac{v}{N}) + \cos(2\pi\frac{u}{M})]$$

4) Since the Gaussian low pass filter with $\sigma = 1$ has a the transfer function of

$H_{lp}(u,v) = A\exp[\frac{-(u^2+v^2)}{2}]$ from Formula sheet→ $h_{lp}(x,y) = A\,(2\pi)e^{-\frac{(2\pi)^2\,(x^2+y^2)}{2}}$

$$H_{hp} = 1 - H_{lp} \Rightarrow h_{hp} = F^{-1}(1-H_{lp}) \Rightarrow h_{hp}(x,y) = \delta(0,0) - A\,(2\pi)e^{\frac{-(2\pi)^2\,(x^2+y^2)}{2}}$$

5)

a)

$$f(x,y) = \cos(4\pi x)\times \sin(8\pi y)$$

$$f(x,y) = \frac{1}{2}[\sin(4\pi x + 8\pi y) - \sin(4\pi x - 8\pi y)] = \frac{1}{2}[\sin\ 2\pi(2x+4y) - \sin 2\pi(2x-4y)]$$

$$F(u,v) = \frac{1}{4j}[\delta(u-2,v-4) - \delta(u+2,v+4) - \delta(u-2,v+4) + \delta(u+2,v-4)]$$

b) $\Delta x = \Delta y = \Delta = \frac{1}{6}$, and $f_s = \frac{1}{\Delta}$, $\Rightarrow f_s = 6\ Hz$ (see Fig. b)

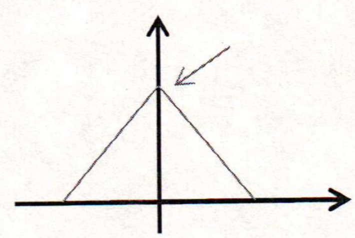
C) $f_s\ /2 = 3\ Hz \Rightarrow -3 < u,v < 3$ (see Fig. c)

So there are one pairs of samples and so

$$F_{r1}(u,v) = \frac{\Delta^2}{4j}[\delta(u-2,v+2) - \delta(u+2,v-2) - \delta(u-2,v-4) + \delta(u+2,v+4)]$$

$$f_{r1}(x,y) = \frac{\Delta^2}{2}[\sin\ 2\pi(2x-2y) - \sin\ 2\pi(2x+2y)] = -\Delta^2 \cos\ (4\pi x)\times \sin\ (4\pi y)$$

d) This is not part of the assignment, but if you want to try another filter as defined below, the result is shown in Fig. d, and the reconstructed signal is calculated below.



$$H_x(u) = \begin{cases} \frac{1}{f_s^2}u + \frac{1}{f_s} & -f_s < u < 0 \\ \frac{-1}{f_s^2}u + \frac{1}{f_s} & 0 < u < f_s \\ 0 & otherwise \end{cases}$$

$$\Rightarrow H_x(u) = \begin{cases} \frac{-1}{f_s^2}|u| + \frac{1}{f_s} & -f_s < u < f_s \\ 0 & otherwise \end{cases}$$

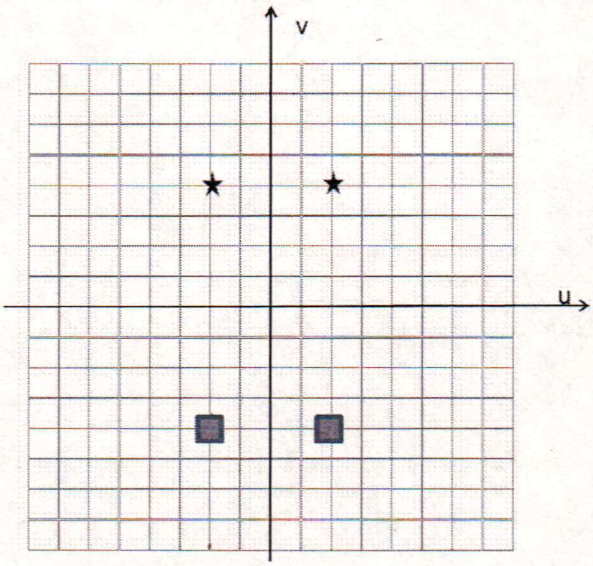
$$H_2(u,v) = H_x(u)\times H_x(v) = \begin{cases} (\frac{-1}{f_s^2}|u| + \frac{1}{f_s})(\frac{-1}{f_s^2}|v| + \frac{1}{f_s}) & -f_s < u\ v < f_s \\ 0 & otherwise \end{cases}$$

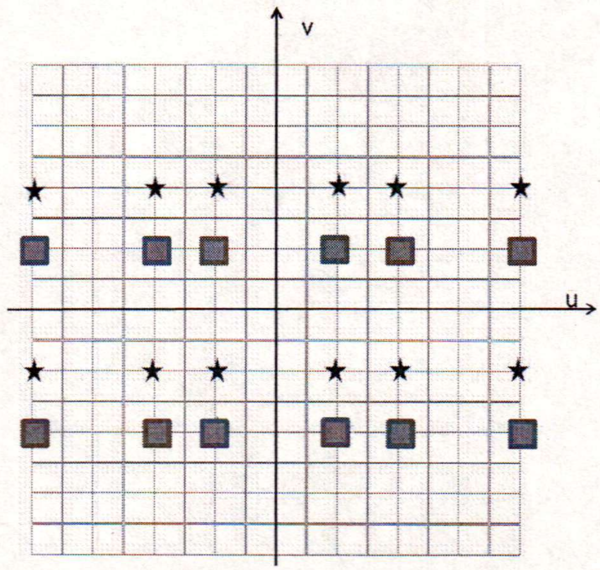
So there are 8 Paris $-6 < u,v < 6$

$$H_2(u,v) = \Delta^4(6-|u|)(6-|v|)$$
$$H_2(|2|,|2|) = 16\Delta^4$$
$$H_2(|4|,|4|) = 4\Delta^4$$
$$H_2(|2|,|4|) = H_2(|4|,|2|) = 8\Delta^4$$

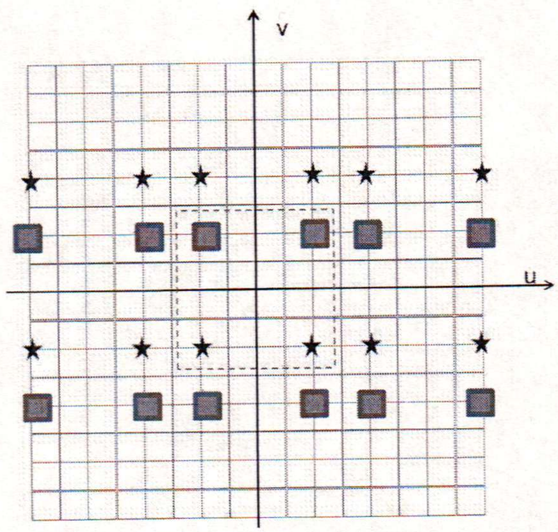
$$F_{r2}(u,v) = H_2(u,v) \times \frac{1}{4j}[\delta(u-2,v+2) - \delta(u+2,v-2) - \delta(u-2,v-2) +$$
$$\delta(u+2,v+2) - \delta(u-4,v+4) + \delta(u+4,v-4) + \delta(u-4,v-4) - \delta(u+4,v+4) +$$
$$\delta(u-2,v-4) - \delta(u+2,v+4) + \delta(u+2,v-4) - \delta(u-2,v+4) + \delta(u-4,v-2) +$$
$$\delta(u+4,v+2) + \delta(u-4,v+2) - \delta(u+4,v-2)]$$

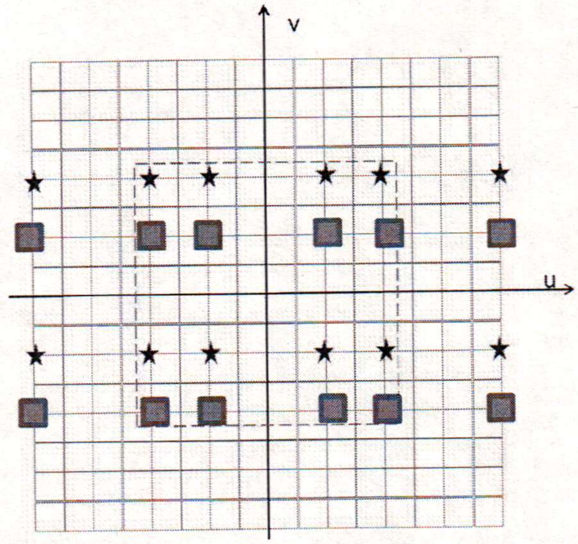
$$f_{r2}(x,y) = \frac{1}{2}[H_2(2,-2)\sin 2\pi(2x-2y) - H_2(2,2)\sin 2\pi(2x+2y) -$$
$$H_2(4,-4)\sin 2\pi(4x-4y) + H_2(4,4)\sin 2\pi(4x+4y) +$$
$$H_2(2,4)\sin 2\pi(2x+4y) + H_2(-2,4)\sin 2\pi(-2x+4y) -$$
$$H_2(4,2)\sin 2\pi(4x+2y) + H_2(4,-2)\sin 2\pi(4x-2y)]$$
$$f_{r2}(x,y) = \frac{\Delta^2}{2}\left(\frac{-4}{9}\cos(4\pi x)\sin(4\pi y) + \frac{1}{9}\cos(8\pi x)\sin(8\pi y) +\right.$$
$$\left.\frac{2}{9}\cos(4\pi x)\sin(8\pi y) - \frac{2}{9}\cos(8\pi x)\sin(4\pi y)\right)$$





(a) (b)





(C) (d)