Assignment #2

1)

$$s = T(r) = (L-1)\int_0^r p_r(w)\ dw = (L-1)\int_0^r Ae^{-w}\ dw = -A(L-1)e^{-w}\Big|_0^r = A(L-1)(1-e^{-r})$$

$$G(z) = (L-1)\int_0^z p_z(w)\ dw = (L-1)\int_0^z B\ w\ e^{-w^2}\ dw = \frac{B}{2}(L-1)\int_0^z e^{-w^2}\ dw = \frac{-B}{2}(L-1)e^{-w^2}\Big|_0^z$$

$$G(z) = \frac{B(1-e^{-z^2})}{2}(L-1)$$

$$s = G(z) \Rightarrow z = G^{-1}(s)$$

$$\frac{(1-e^{-z^2})}{2} = \frac{A}{B}(1-e^{-r}) \quad \Rightarrow e^{-z^2} = 1 - \frac{2A}{B}(1-e^{-r}) \Rightarrow -z^2 = \ln[1 - \frac{2A}{B}(1-e^{-r})]$$

$$z = \sqrt{-\ln[1-\frac{2A}{B}(1-e^{-r})]}$$

2)

Calculate Histogram equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)\ dw = (L-1)\int_0^r (-2w+2)\ dw = (L-1)(-r^2+2r)$$

$$G(z) = (L-1)\int_0^z p_z(w)\ dw = (L-1)\int_0^z 2w\ dw = (L-1)z^2$$

$$s = G(z) \Rightarrow z = G^{-1}(s)$$

$$z = \sqrt{-r^2+2r}$$

3)

If variables r and s are continuous values, we can produce flat histograms. However, the intensity values of a digital image are discrete. In the continuous domain, there are infinite numbers of values in any interval [r, r+dr]. In digital images, we have only a finite number of pixels in each range. As the range is stretched, and the number of pixel in this range is preserved, there are only a finite number of pixels with which the stretched range is populated. The histogram that results is spread over the whole range of gray values, but it is far from flat.

4)

(a) The image has 50x70 = 3500 pixels. Each pixel has values from 0..7. The counts follow the relationship H(r) = kr. To determine k, we know that the sum of the values of H(r) for all r must sum to 3500. $\sum_{r=0}^{7} Kr = 3500 \Rightarrow K(0+1+2+3+4+5+6+7) = 3500$

You can directly evaluate the sum in parentheses, or use the fact that 1+2+…+*n* = *n*(*n*+1)/2

Therefore k(7)(8)/2 = 3500, or 28k =3500. Or k = 3500/28 = 125

(b) We compute the histogram using H(r) = 125r. We compute the probability density function using p(r) = H(r)/3500:

r	H(r)	P(r)
0	0	0
1	125	0.0375
2	250	0.0714
3	375	0.1071
4	500	0.1429
5	625	0.1786
6	750	0.2143
7	875	0.25
sum	3500	1

The mean is $\mu = \sum_{r=0}^{7} r\, p_r(r) = 5$ The standard deviation is $\sigma = \sqrt{\sum_{r=0}^{7} (r-\mu)^2 p_r(r)} = 1.73$

r	H(r)	P(r)	r p(r)	P(r)*(r-mean)^2
0	0	0	0	0
1	125	0.0375	0.0375	0.6
2	250	0.0714	0.1428	0.6426
3	375	0.1071	0.3213	0.4284
4	500	0.1429	0.5716	0.1429
5	625	0.1786	0.893	0
6	750	0.2143	1.2858	0.2143
7	875	0.25	1.75	1
sum	3500	1	5.002	3.0282

(c)

r	H(r)	P(r)	S=T(r)
0	0	0	0
1	125	0.0375	0
2	250	0.0714	1
3	375	0.1071	2
4	500	0.1429	3
5	625	0.1786	4
6	750	0.2143	5
7	875	0.25	7

S	H(s)
0	125
1	250
2	375
3	500
4	625
5	750
6	0
7	875
sum	3500

The histogram of the transformed image is given here. Note that the counts in the new histogram still sum to 3500.

e) Use the same equations as in (b) for the mean and standard deviation:

s	H(s)	P(s)	s p(s)	P(s)*(s-mean)^2
0	125	0.0375	0	0.677407502
1	250	0.0714	0.0714	0.754255323
2	375	0.1071	0.2142	0.542290144
3	500	0.1429	0.4287	0.223352706
4	625	0.1786	0.7144	0.011180367
5	750	0.2143	1.0715	0.120479469
6	0	0	0	0
7	875	0.25	1.75	1.89035001
sum	3500	1	4.2502	4.21931552

$\sigma = \sqrt{4.2193}$, $\mu = 4.2502$. The mean of the enhanced image is closer to the middle of the gray level range, and the standard deviation is larger, as we would expect.