ELEX 7720: Feedback Controls

Lab 2 and 3 – Server Motor Modelling and Controlling

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Introduction

Servomotors are used to track a specific trajectory in either rotational or translational motion. Over the course of this lab, we used a Faulahber DC motor. The motor's motion was geared down using an external load disk. A tachometer is also attached to the whole apparatus and used to collect velocity data from the motor. An encoder is in place to collect the rotational position of the motor. Data is acquired using a USB DAQ running at a fixed sampling rate. SIMULINK and Quarc are used as the frontend interface for the motor.

Transfer Function Modelling

The model parameters tabulated in Figure 1 and the Servo Motor specific information displayed in Figure 2 were used to obtain the first and second order models. Equations pertaining to the electrical and mechanical properties of the Servo Motor were gleaned from ELEX7720 Lab1 – Commissioning the Servo Motor Control System. The parameters for the second order equations were also obtained from this document.

Parameter	Numerical Value	SI Units
Motor Armature Resistance R_m	2.6	Ω
Motor Armature Inductance L_m	0.18	mH
Motor Efficiency η_m	0.69	
Motor Torque Constant k_t	7.68e-3	$\frac{N \cdot m}{amp}$
Back EMF k_m	7.68e-3	$\frac{V \cdot \sec}{rad}$
Motor Shaft Moment of Inertia J_m	3.9e-7	$kg \cdot m^2$
Total Gear Inertia J_l	1.08e-5	$kg \cdot m^2$
Motor Viscous Damping Coefficient B_m	8.149e-7	$\frac{N \cdot m \cdot \sec}{rad}$
Internal Gear Ratio K_{gi}	14	
the external motor gear has $N_3 = 72$ teeth		
the load gear has $N_4 = 72$ teeth		
External Gear Ratio K_{ge}	1	
Total Gear Ratio K_g	14	
Gearbox Efficiency η_g	0.90	
Moment of Inertia on Motor Side J_{eq}	4.514e-7	$kg \cdot m^2$
Viscous Damping on Motor Side B_{eq}	8.149e-7	$\frac{N \cdot m \cdot \sec}{rad}$
Amplifier Gain K_a	1	V / V
Motor Transfer Function:	$G_{m,2nd}(s) =$ (6.522e7)/((s^2) + 1.445e4s + 5.27e5) $G_{m,1st}(s) =$ 123.8/(1 + 0.0274s)	

Figure 1: Model Parameters

Nominal input voltage	6.0 V
Motor Armature Resistance	2.6 Ω
Motor Armature Inductance	0.18 mH
Motor Torque Constant	7.68e-3 N-m/amp
No Load Speed	7200 RPM
No Load Current (shaft diameter is 0.12”)	0.08 amps
Motor Efficiency	0.69
Back EMF Constant	7.68e-3 N-m
Motor Shaft Moment of Inertia	3.9e-7 kg-m^2
Internal Gearbox Ratio	14
Motor Gear	72 teeth
Load Gear	72 teeth
Gearbox Efficiency	0.90
mass of the 72 tooth gear	0.03 kg
radius of the 72 tooth gear	0.019 m
tachometer sensitivity (measuring **motor** speed) [6]	1.5mV/rpm
Encoder	360 deg (counter-clockwise) /4096 counts

Figure 2: Servo Motor Specifications

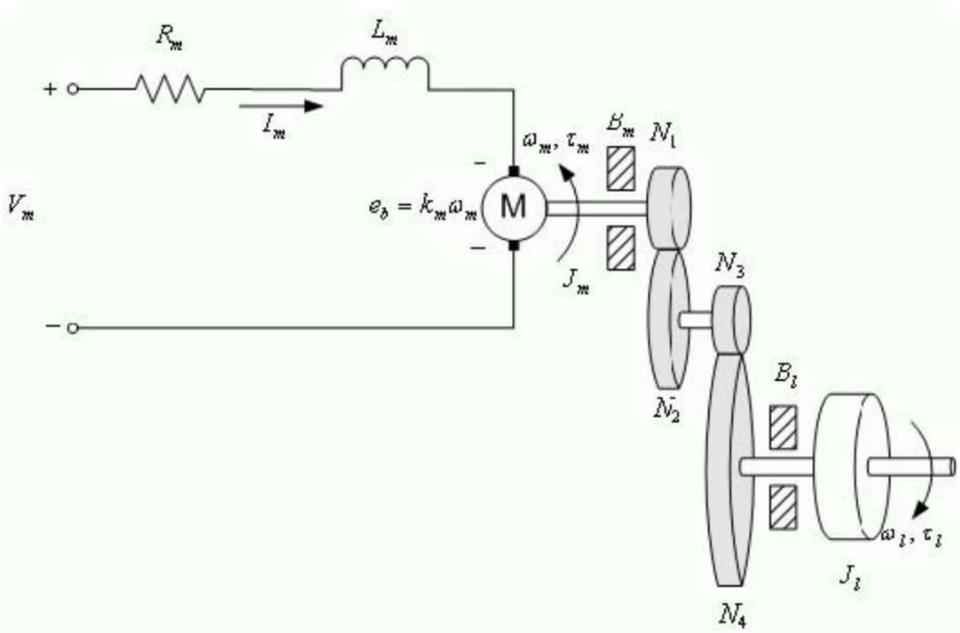


Figure 3: Schematic of Servo Motor

Figure 3 shows the mechanical and physical aspects of the armature controlled DC motor. The armature voltage is the input to the system while the output is the angular position and velocity of the motor shaft. The symbols of the components in Figure 3 are explained in Figure 1.

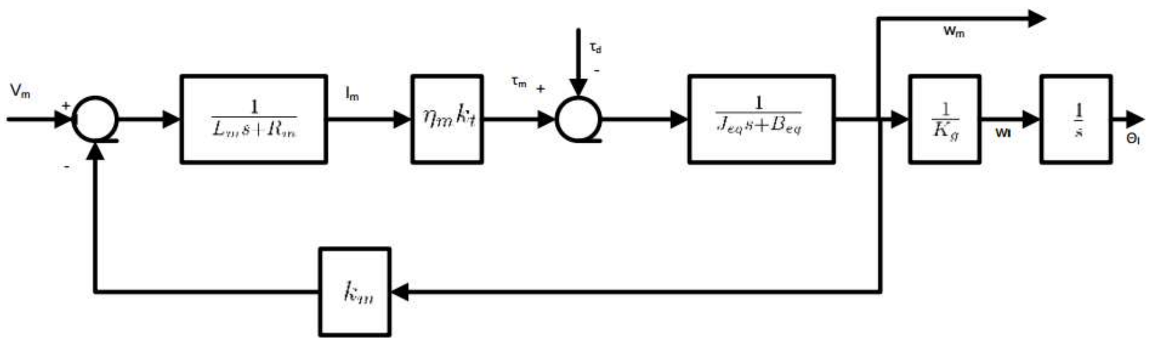


Figure 4: Block Diagram of the DC Motor

The second order model was derived by collapsing the block diagram in Figure 4 and then comparing to the second order transfer function format shown below:

$$G_{m,2nd}(s) = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The result of the comparisons allowed as to identify the parameters as follows:

$$K = \frac{\eta_m k_t}{L_m J_{eq}},\ \ 2\zeta\omega_n = \frac{B_{eq}L_m+J_{eq}R_m}{L_m J_{eq}}, \text{ and } \omega_n^2 = \frac{R_m B_{eq}+\eta_m k_t k_m}{L_m J_{eq}}$$

The calculations for the equivalent moment of inertia J_{eq} and viscous damping B_{eq} on the motor side are shown below:

$$J_{eq} = \frac{J_l}{\eta_g K_g^2} + J_m$$

$$J_{eq} = \frac{1.08e-5\ kg.m^2}{0.69(14)} + 3.9e-7kg.m^2 = 4.514e-7kg.m^2$$

$$B_{eq} = \frac{B_l}{\eta_g K_g^2} + B_m$$

We assume that the viscous load coefficient is approximately zero leading B_{eq} to be equal to $B_m$or the motor viscous damping coefficient.

$$B_{eq} = B_m = 8.149e-7\ \left(\frac{N.m.sec}{rad}\right)$$

The total gear ratio K_g is the product of the external and internal gear ratio which is $\frac{N_4}{N_3}$ and $\frac{N_2}{N_1}$ respectively.

The parameters for the second order model can now be calculated as follows:

$$K = \frac{\eta_m k_t}{L_m J_{eq}}$$

$$K = \frac{(0.69)7.68e-3}{0.18(4.514e-7)} = 6.522e7$$

$$2\zeta\omega_n = \frac{B_{eq} L_m + J_{eq} R_m}{L_m J_{eq}}$$

$$2\zeta\omega_n = \frac{(8.149e-7)(0.18)+(4.514e-\ \ )(2.6)}{0.18(4.514e-7)} = 1.445e4$$

$$\omega_n^2 = \frac{R_m B_{eq} + \eta_m k_t k_m}{L_m J_{eq}}$$

$$\omega_n^2 = \frac{(2.6)(8.149e-7) + (0.69)(7.68e-3)(7.68e-3)}{0.18(4.514e-7)} = 5.27e5$$

If we assume that the motor inductance is much less than the armature resistance then the second order model can be reduced to the form of the first order model shown below.

$$G_m(s) \approx \frac{K}{\tau_s + 1}$$

The first order model parameters can be described by the equations below:

$$K = \frac{\eta_m k_t}{R_m B_{eq} + \eta_m k_t k_m}, \tau = \frac{R_m J_{eq}}{R_m B_{eq} + \eta_m k_t k_m}$$

The gain, K and the time constant, τ were calculated to be as follows:

$$K = \frac{(0.69)(7.68e-3)}{(2.6)(8.149e-7) + (0.69)(7.68e-3)^2} = 123.8$$

$$\tau = \frac{(2.6)(4.514e-7)}{(2.6)(8.149e-7) + (0.69)(7.68e-3)^2} = 0.0274$$

The encoder calibration was achieved using the gain formula below:

$$K_{enccal} = -\frac{1}{4096counts}(360°)$$

This was derived from the fact that the encoder was set for 360 deg for every 4096 counts as can be seen in Figure 2.

The tachometer sensitivity according to Figure 2 was 1.5mv per rpm which lead to the tachometer calibration to be as follows:

$$K_{tachcal} = \frac{360}{\left(60(K_g)(1.5e-3\,V)\right)}$$

Using all of these we can build our model system in Simulink.

Modeling the system, we used and built a subsystem using the second order model that was established in Figure 1: model parameters. This system can be

Dead Band Adjustment

The dead band is an irregularity in the way that the motor functions, where the striction of the motor prior to moving. Our motor controller transfer function doesn't deal with this irregularity as it is a non-linear behavior. However outside of the controller model itself, we are able to build a dead zone block that will help our model more closely follow the physical response of the system.

Without accounting for the dead band in this servo we can see that the velocity response of the motor is not quite what our model predicts. This is due to the dead band area where the output does not respond to the changing input signal.

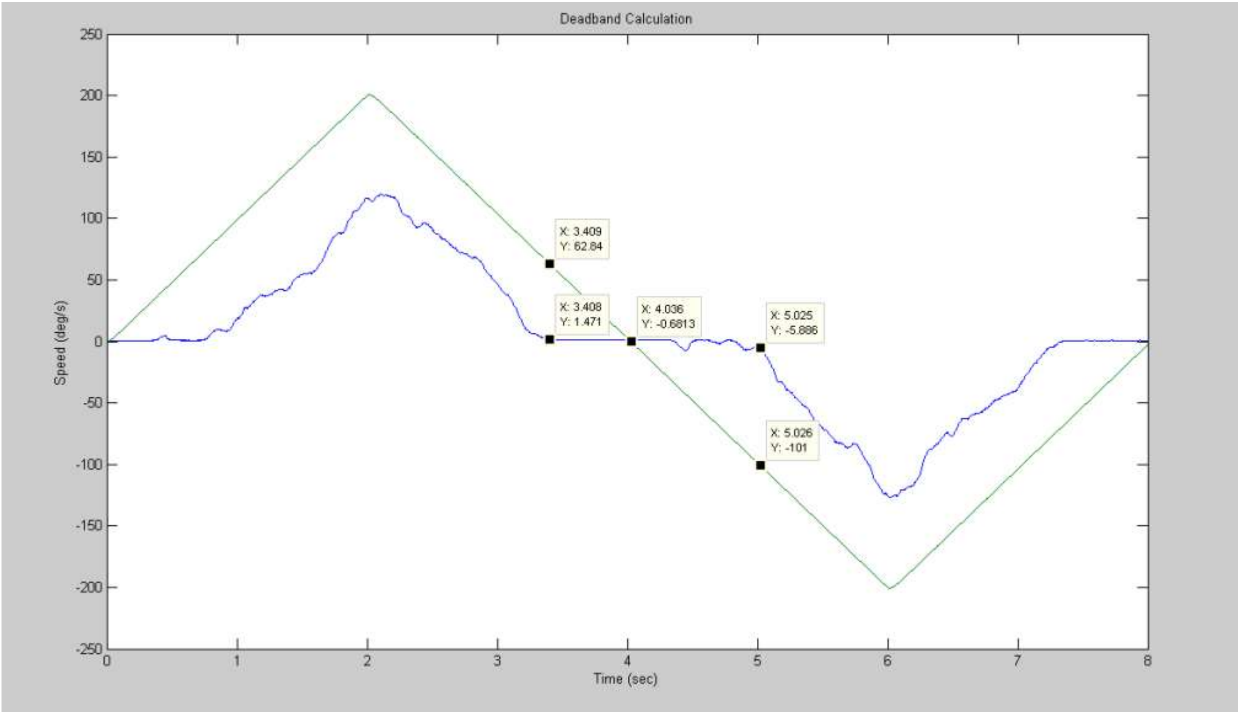


Figure 5Dead with points of interest

The deadband acts as a permanent arithmetic subtraction on the absolute value of the velocity. As can be seen in the graph it not only causes a lag in the emotion of the motor, but also shows that the intensity of the output is greatly diminished.

The five point that we noted were the points needed to calculate the exact dead zone. The equation goes as

$$db = \frac{\pi}{180}\left(\frac{K_g}{K_m}\right)\left(\frac{155.4 + 166.8}{2}\right) = 0.32\ V$$

However we can also directly get the deadband offset voltage by simply looking up the voltage that causes the armature to start moving:

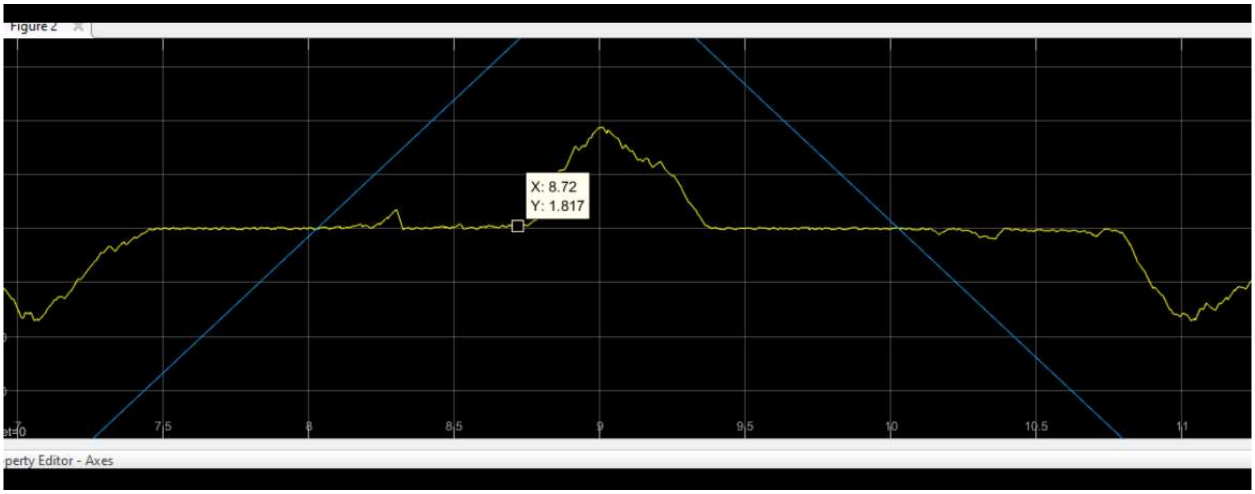
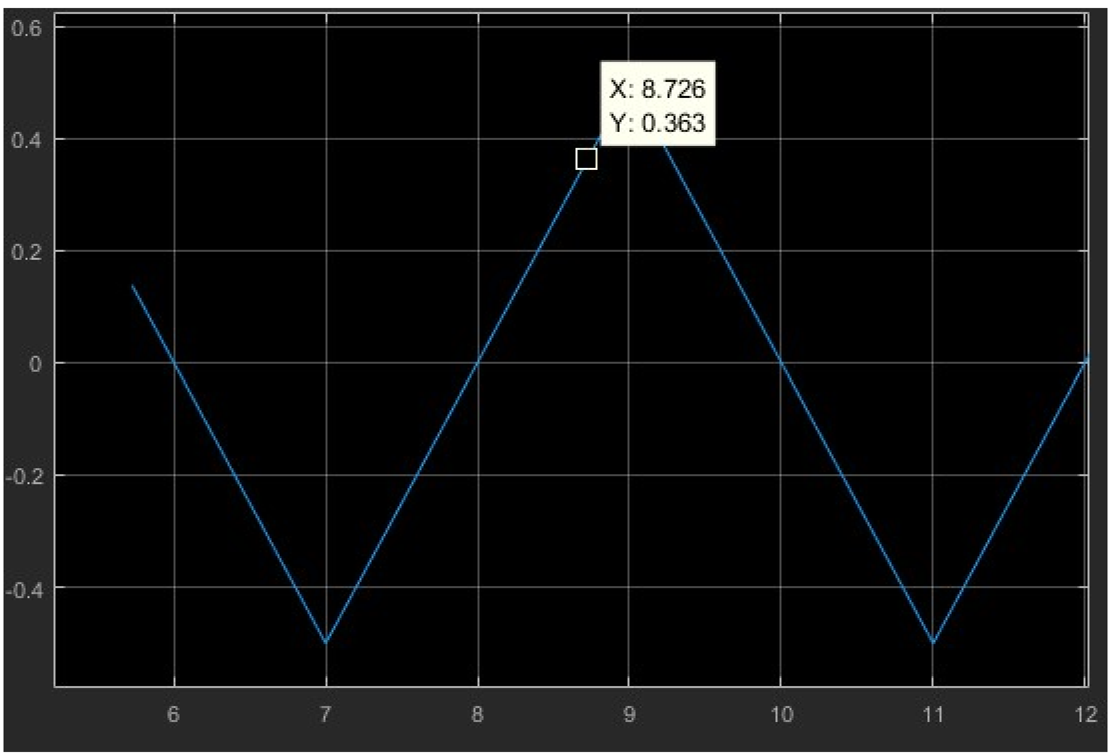
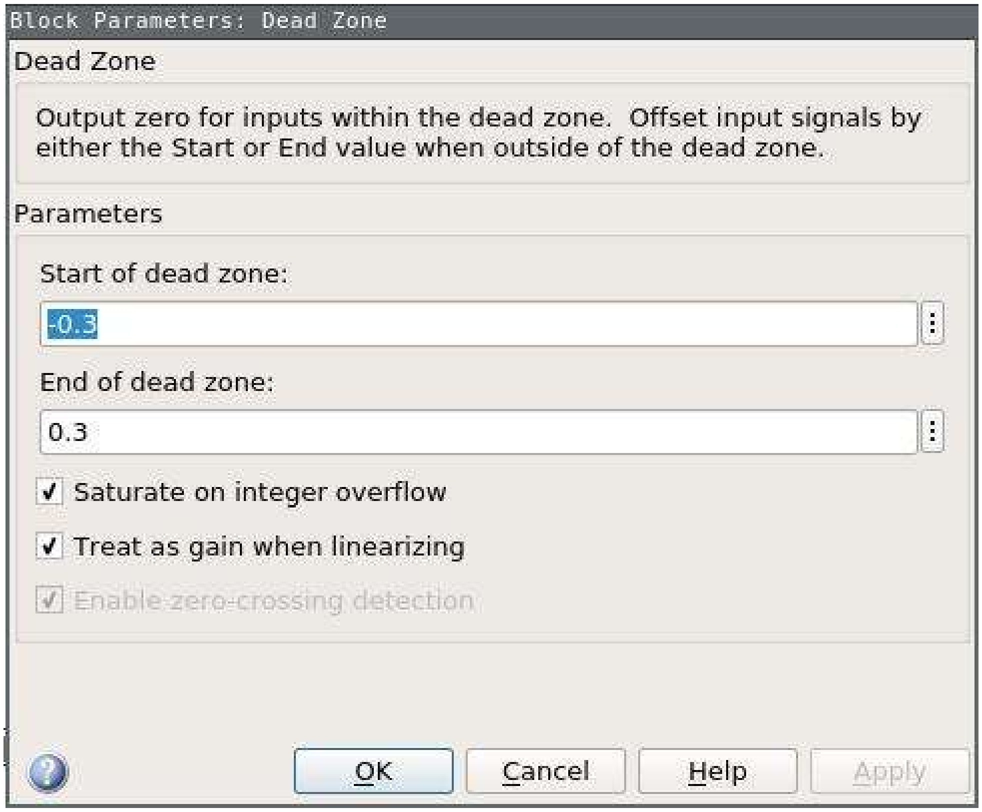


Figure 6 Deadband in the actual system

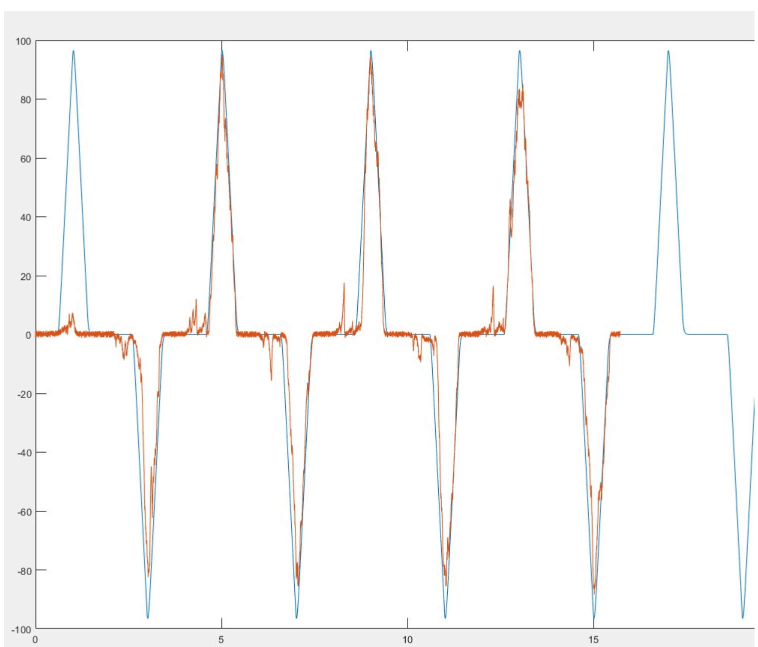
From Fig 6 we can see that the time that the dead band ends is at 8.72 seconds, looking this up in our historical data for the armature voltage,



We can see that the voltage in this case is about 0.363 volts, which is quite close to the calculated value that we saw earlier. Once we had the deadzone and set it in the model parameters,



Rerunning the simulation with the proper dead zone modeling we can see that there is marked improvement,



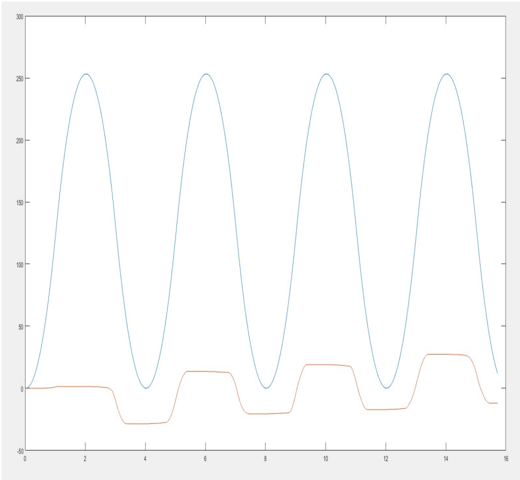
We can see that in this case the system matches the responses quite closely.

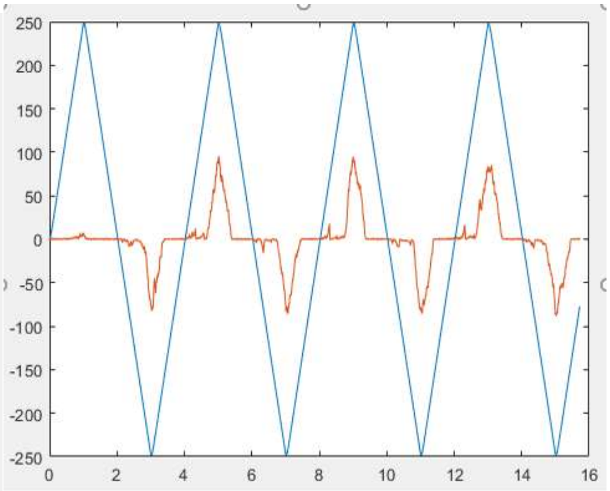
Model Performance

We didn’t change transfer function gain or time constants we just took those from the parameters given to us from the datasheets and used them as is. Additionally, we only modeled our system with a second order transfer function, while the real system may have some higher-order dynamics that may be present in the actual physical system. One more point of performance that we didn’t account for was the presence of noise and disturbance. This means that there are still errors that are present in the systems that we model. But the error for most cases are actually quite minor.

Triangle Wave

Here are the angle and velocity reponses of the triangle wave





As you can see in this case without accounting for the dead zone it’s quite bad but it was at this stage, we hadn’t applied the deadzone yet, you can see can see how the dead causes zeroes and no-response areas in velocity, and causes attenuation and flattening in the position, we can also see a bit of drift as a result of the deadzone not being symmetric about both sides.

After applying the dead zone to the model.

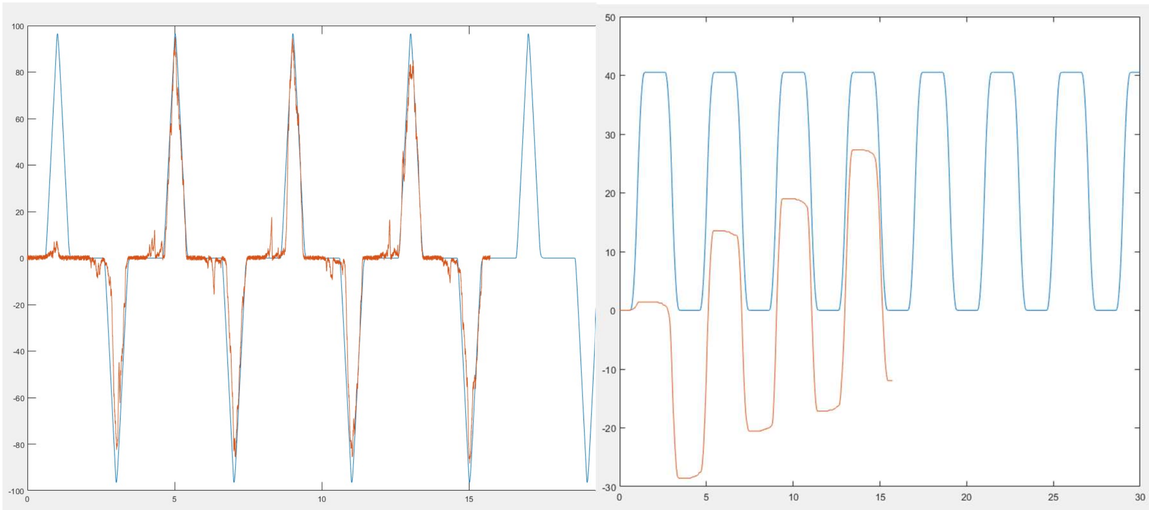
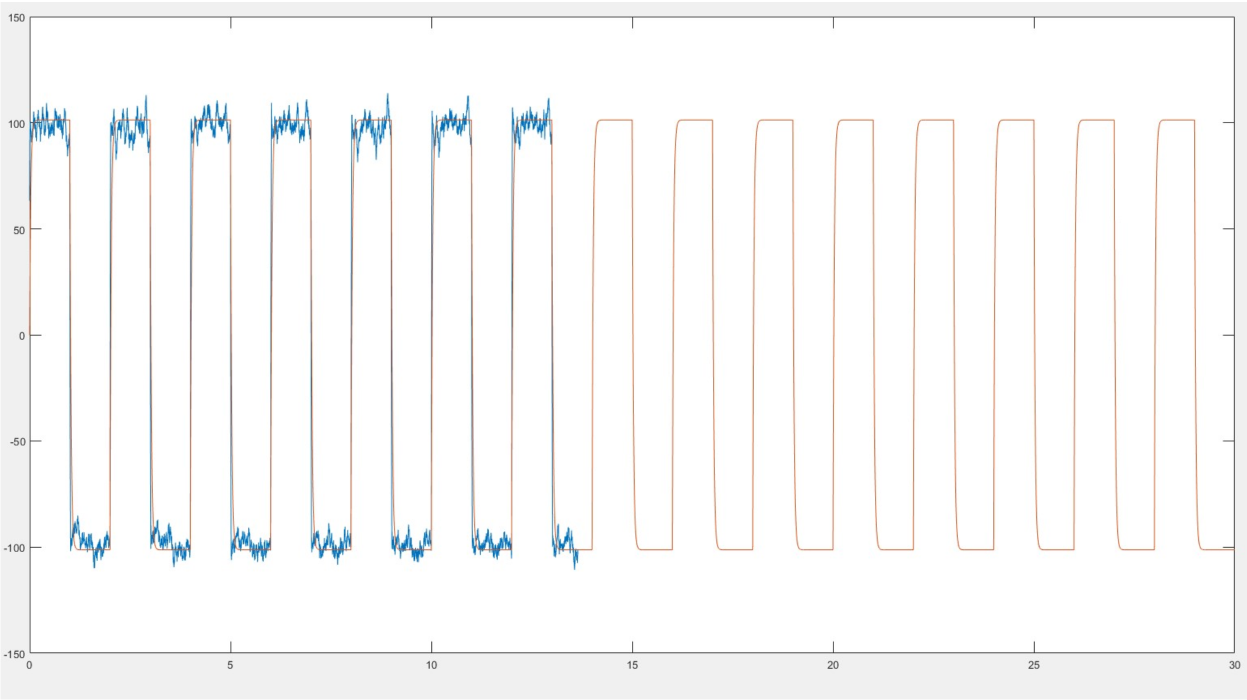


Figure 7Velocity Left and Theta Right

With the dead zone in the model we can see that the model matches much more closely, there is a bit of transience at the beginning of the actual motor's start point that causes it to stick and move in one direction. Note that our model doesn't drift because we had a symmetric dead zone defined in our model

Velocity RMS error = 2.291

Square Wave



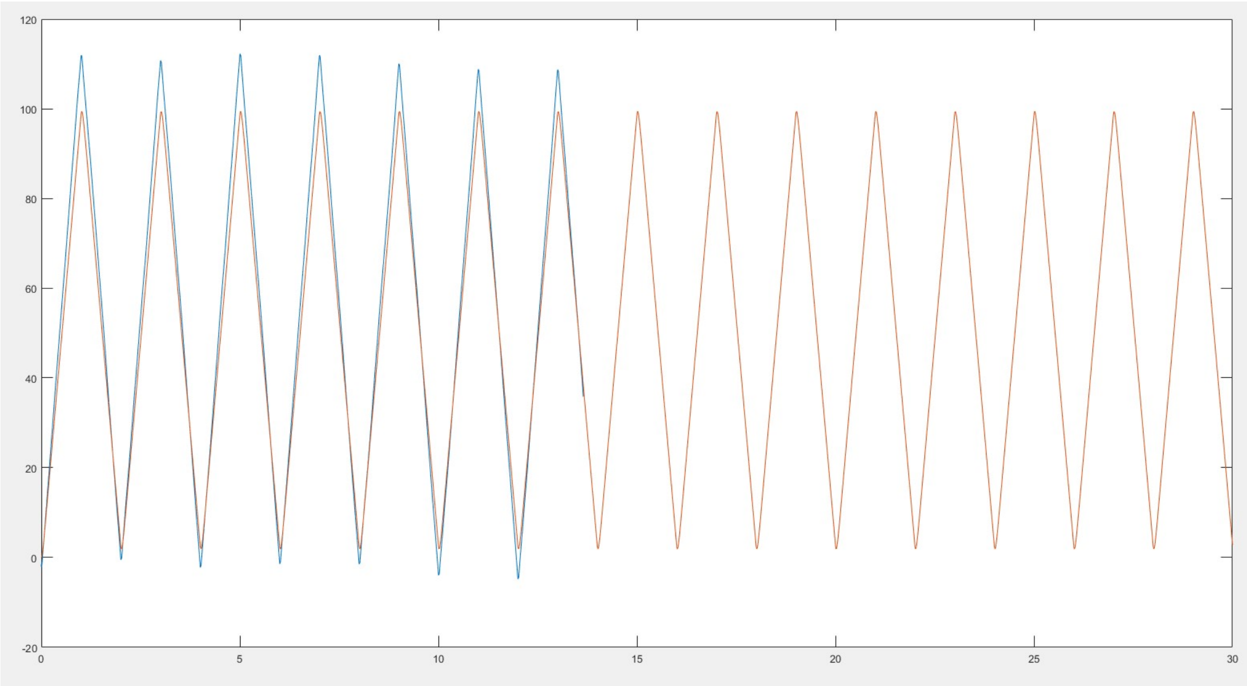


Figure 8 Velocity Top, Angle bottom.

The square wave shows very good response that resembles the actual motor characteristics extremely closely.

Velocity RMS error = 0.237

Conclusion

Over the course of this lab we set up a motor and built a controller to model the responses of this motor. By carefully studying the mechanics of the motor, and the responses of the motors to electronics, we were able to build a preliminary second order model to have a starting point for studying the motor's characteristics. By comparing the responses of our model to responses of the actual motor, we were able to resolve non-linearities in the operation of the motor caused by the static friction of the moving mechanical parts. Once that was done we were left with a very realistic model of the actual motor's responses to a variety of different inputs. This leaves us with an excellent starting point for the development of a controller system for controlling the motor's movement.